→ The important characteristic of a VECTOR quantity is that it has both magnitude (or size) and a direction.

→ Things to be learned :
- a) how to ~~use~~ write down vectors.
- b) how to add and subtract them,
- c) how to use them in geometry.

→ A VECTOR is a quantity with magnitude (or length) and a direction.

Example: velocity, acceleration, momentum, force, electric and magnetic field intensities.

→ The direction of the vector is indicated by an arrow pointing from the tail to the head.

If the tail is at point A and head is at point B,

point A $\bullet\longrightarrow\bullet$ point B : $\overrightarrow{AB}$

∴ The vector from A to B is written $\overrightarrow{AB}$.

· The length is always a non-negative real number and length (magnitude) of a vector v is written $|v|$.

→ Two vectors are equivalent if they have the same magnitude and direction.

→ The length or magnitude of $\overrightarrow{AB}$ is expressed as $|\overrightarrow{AB}|$.

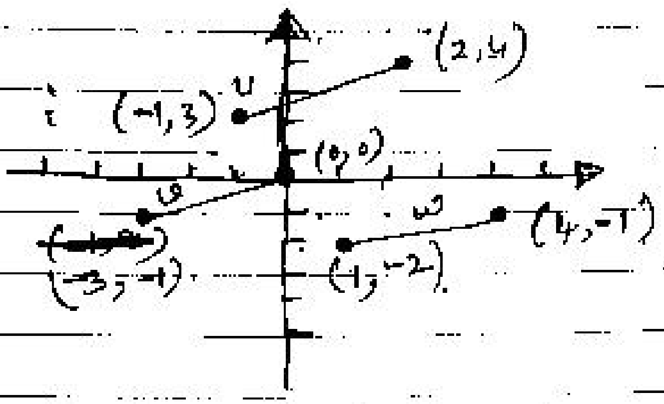
→ A SCALAR is a real number ;

Examples of Scalars

Mass, volume, temperature, density, electric charge, work etc.

P.T.O.

Example :-



length of each vector by using the distance formula.

$$|u| = \sqrt{\{2-(-1)\}^2 + (4-3)^2} = \sqrt{9+1} = \sqrt{10}.$$

$$|v| = \sqrt{[0-(-3)]^2 + (0-(-1))^2} = \sqrt{9+1} = \sqrt{10}$$

$$|w| = \sqrt{(4-1)^2 + [-1-(-2)]^2} = \sqrt{9+1} = \sqrt{10}$$

Here, all vector u, v, w are shown in the same platform.

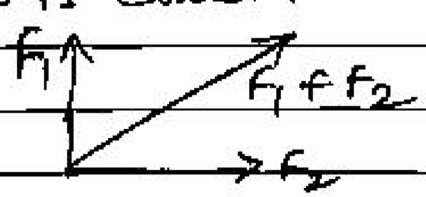
$$\therefore \text{slope}, P = \frac{4-3}{2-(-1)} \quad \frac{0-(-1)}{(0-3)} \quad \frac{-1-(-2)}{4-1}$$

$$= \frac{1}{3}$$

All three vectors are same magnitude & direction

⇒ Direction cosine refers to the cosine of the angle between any two vectors.

⇒ The sum of the two vectors is called the ~~resultant~~.



~~If $f_1 = 15$ & $f_2 = 25$~~

~~then $|v| = 15^2 + 25^2$~~

$$\therefore v = \sqrt{15^2 + 25^2} \qquad \tan\theta = \frac{15}{25} = 0.6$$

$$|v| = 29.2 \qquad \theta = \tan^{-1}(0.6) = 31^\circ.$$

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B-4, U-1, Vector-1.

SPIRX

No.

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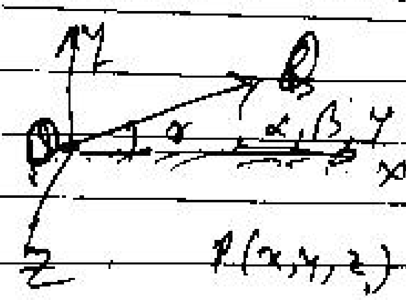
Q Prove that $l^2 + m^2 + n^2 = 1$, where l, m and n are direction of a vector.

Sol $l = \cos\alpha$, $m = \cos\beta$, $n = \cos\gamma$



$x = r\cos\alpha$

$y = r\cos\beta$

$z = r\cos\gamma$ and, $r = |r| = \sqrt{x^2+y^2+z^2}$

$\cos\alpha = \frac{x}{r}$; $\cos\beta = \frac{y}{r}$; $\cos\gamma = \frac{z}{r}$.

$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma = \frac{x^2}{r^2} + \frac{y^2}{r^2} + \frac{z^2}{r^2}$

$= \frac{x^2+y^2+z^2}{r^2} = \frac{r^2}{r^2} = 1$

Q Find the M & D of the position vector of point $P(1, -1, \sqrt{2})$

Sol $|OP| = r$

$r = \sqrt{x^2+y^2+z^2} = \sqrt{1^2+(-1)^2+\sqrt{2}^2}$

$= \sqrt{4} = 2$

Now, $\cos\alpha = \frac{x}{r} = \frac{1}{2}$

So that, $\alpha = \frac{\pi}{3}$, $\cos\beta = \frac{y}{r} = -\frac{1}{2}$

So, $\beta = \frac{2\pi}{3}$ & $\cos\gamma = \frac{z}{r} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

So that $\gamma = \frac{\pi}{4}$.

⇒ Q. If the position vector of the point $P(x, 0, 3)$ has magnitude 5, find the value of x

Sol. $|OP| = r = 5$

$\therefore |\vec{OP}| = \sqrt{x^2+y^2+z^2}$ $\Rightarrow 5 = \sqrt{x^2+0^2+3^2}$

or, $25 = x^2+9$ $\therefore$

or, $x^2 = 25-9 = 16 = \pm 4$.

⇒ Q Let $\vec{a} = 4\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{b} = -2\hat{i} + 5\hat{j} + 3\hat{k}$

find the component from a magnitude of the following vectors

(a) $\bar{a} + \bar{b}$

Add the value of $\vec{a}$ and $\vec{b} = 2\hat{i} + 2\hat{j} + 4\hat{k}$

So, $|a+b| = \sqrt{2^2+2^2+4^2} = \sqrt{24} = 2\sqrt{6}$.

(b) $\vec{a} - \vec{b}$ using values.

$\vec{a} - \vec{b} = 6\hat{i} - 8\hat{j} - 2\hat{k}$

$|\vec{a} - \vec{b}| = \sqrt{6^2 + (-8)^2 + (-2)^2} = \sqrt{104} = 2\sqrt{26}$.

(c) $\frac{5}{2}\vec{a}$ By using the given values, we can get

$\frac{5}{2}\vec{a} = \frac{5}{2}(4\hat{i} + 3\hat{j} + \hat{k}) = 10\hat{i} + \frac{15}{2}\hat{j} + \frac{5}{2}\hat{k}$

So that,

$\left|\frac{5}{2}\vec{a}\right| = \sqrt{100 + \frac{225}{4} + \frac{25}{4}} =$

⇒ Q. Let $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + 4\hat{j} - \hat{k}$.

Find (a) $|\vec{a} - \vec{b}|(\vec{a} + \vec{b})$ (b) $|2\vec{a} - 3\vec{b}|$.

Sol By using the values, $\vec{a} + \vec{b} = 3\hat{i} + 2\hat{j} + \hat{k}$.

So that $\overline{OB} + \overline{OB} = \overline{OB}$, Now, $|\bar{a} - \bar{b}| = \sqrt{(-1)^2 + (-6)^2 + 2^2}$
$= \sqrt{46}$

Therefore, $|\vec{a} - \vec{b}|\{\vec{a} + \vec{b}\} = \sqrt{46}(3\hat{i} + 2\hat{j} + \hat{k})$.

(b) By using the given values, we get,

$2\vec{a} = 2\hat{i} - 4\hat{j} + 4\hat{k}$ and $3\vec{b} = 6\hat{i} + 12\hat{j} - 3\hat{k}$

Now $2\vec{a} - 3\vec{b} = -4\hat{i} - 16\hat{j} + 7\hat{k}$

$\therefore |2\vec{a} - 3\vec{b}| = \sqrt{(-4)^2 + (-16)^2 + (7)^2} = \sqrt{321}$.

⇒ Q Find a unit vector in the direction of $\vec{a}+\vec{b}$ where $\vec{a} = 2\hat{i} + 2\hat{j} - 5\hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$

Sol

By using the given values, we can get

$$\vec{a}+\vec{b} = 4\hat{i} + 3\hat{j} - 2\hat{k}$$

$$|\vec{a}+\vec{b}| = \sqrt{4^2+3^2+(-2)^2} = \sqrt{29}$$

∴ Unit vector in the direction of a+b

$$= \frac{a+b}{|\vec{a}+\vec{b}|} = \frac{1}{\sqrt{29}}(4\hat{i} + 3\hat{j} - 2\hat{k}).$$

⇒ Q Find a unit vector in the direction of vector $\vec{PQ}$ joining the points P (1, 2, 3) and Q (−1, 1, 2).

Sol Here, $\vec{PQ} = (-1-1)\hat{i} + (1-2)\hat{j} + (2-3)\hat{k}$

$$= -2\hat{i} - \hat{j} - \hat{k}.$$

$$\therefore |\vec{PQ}| = \sqrt{(-2)^2+(-1)^2+(-1)^2} = \sqrt{6}.$$

∴ Unit vector in direction of $\vec{PQ}$

$$= \frac{\vec{PQ}}{|\vec{PQ}|} = \frac{1}{\sqrt{6}}(-2\hat{i} - \hat{j} - \hat{k})$$

⇒ Q Show that vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $4\hat{i} + 6\hat{j} - 2\hat{k}$ are collinear.

Sol Let $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$. and $\vec{b} = 4\hat{i} + 6\hat{j} - 2\hat{k}$

So that $\vec{b} = 2(2\hat{i} + 3\hat{j} - \hat{k}) = 2\vec{a}$

Now, it is clear that $\vec{b}$ is a scalar multiple of a.

Therefore, a and b are collinear vectors.

P.T.O

SPIRX

No.

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Q Show that the three points $A(6, -7, -1)$; $B(2, -3, 1)$ and $C(4, -5, 0)$ are collinear.

Sol

We have,

$\vec{AB} = (2-6)i + (-3+7)j + (1+1)K.$

or $\vec{AB} = -4\hat{i} + 4\hat{j} + 2\hat{K}.$

and $\vec{AC} = (4-6)i + (-5+7)j + (0+1)\hat{K}$

or $\vec{AC} = -2i + 2j + \hat{K}$

From both the equation, it is clear that

$\vec{AB} = 2\vec{AC}$

Therefore, $\vec{AB}$ and $\vec{AC}$ are Collinear Vectors and also A, B and C are Collinear points.

Q Find the Unit vector in the direction of the sum of the vectors $\vec{a} = 2i + 2j - 5K$ and $\vec{b} = 2i + \hat{j} + 3K.$

Sol

The vector $\vec{PQ}$, joining the points $P(2, 2, -5)$ and $Q(2, 1, 3)$

By using the given values we can get

$\vec{PQ} = (2-2)i + (1-2)j + (3+5)K$

$= \sqrt{(-1)^2 + 8^2}$

$= \sqrt{1+64} = \sqrt{65}$

SPIRX No. Date 16/07/2021

→ Application of vectors in specific situations ...

→ Things to be learned :-

a) Vector in two and three dimensions.

b) Double and triple scalar and vector Product and their uses

→ It's also known "Dot Product" or "Inner Product".

→ The scalar product of two vectors.

(i) If, $\vec{u}_1 = (a_1, a_2)$, $\vec{u}_2 = (b_1, b_2)$

It's defined as $\vec{u}_1 \cdot \vec{u}_2 = a_1 b_1 + a_2 b_2$

(ii) If $\vec{u}_1 = (a_1, a_2, a_3)$ and $\vec{u}_2 = (b_1, b_2, b_3)$

Then $\vec{u}_1 \cdot \vec{u}_2 = a_1 b_1 + a_2 + b_2 + a_3 b_3$.

→ If $\vec{u} = (a_1, a_2, a_3)$ and $\vec{v} = (b_1, b_2, b_3)$

Then $\vec{u} \cdot \vec{v} = a_1 b_1 + a_2 b_2 + a_3 b_3$

also $\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\theta$ [∵ θ is angle between $\vec{u}$ and $\vec{v}$]

→ $\vec{u} \cdot \vec{u} = |\vec{u}|^2$ if $\vec{u} \cdot \vec{u} \geq 0 \ \forall \vec{u}$.

→ $\vec{u} \cdot \vec{u} = 0$ if $\vec{u} = 0$

→ $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

→ $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

→ $(\alpha \vec{u}) \cdot \vec{v} = \alpha (\vec{u} \cdot \vec{v}) = \vec{u} \cdot (\alpha \vec{v})$. [∵ α is a scalar]

→ Vector (cross) product of Vectors.

** $\vec{a} \times \vec{b} = (|\vec{a}||\vec{b}| \sin\theta) \vec{n}$

∵ when vector $\vec{a} \cdot \vec{b}$ ~~is defined at angle θ,~~ another unit vector $\vec{n}$ are constitute a right ~~orthogonal triad (⊥)~~ Then vector product of $\vec{a}$ & $\vec{b}$ can be written as **

means,
$\vec{n}$ = Unit + Vector ⊥ to both $\vec{a}, \vec{b}$

$\rightarrow A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$

and $(A \times B) \times C = (C \cdot A)B - (C \cdot B)A$.

↑ for evaluating a vector triple product :-

Example

If the vectors $A = 2i + 3j - 4K$,

$B = i - 2j + 2K$,

$C = 3i - 3j - K$

Find the vector triple product, $A \times (B \times C)$?

Soln We know,

$A \times (B \times C) = (A \cdot C)B - (A \cdot B)C$.

$\rightarrow$ 1st calculate: $A \cdot C = 2 \times 3 + 3 \times (-3) - 4 \times (-1) = 1$

$\rightarrow$ 2nd calculate: $A \cdot B = 2 \times 1 + 3(-2) - 4 \times 2 = -12$

Putting the value in formula:

$A \times (B \times C) = 1\,(i - 2j + 2k) - (-12)(3i - 3j - K)$

$= i - 2j + 2K - \{-36i + 36j + 12K\}$

$= i - 2j + 2K + 36i - 36j - 12K$

$= 37i - 38j - 10K$.

Q. If $\vec{a} = 5i - 3j - 3K$ and $\vec{b} = i + j - 5\vec{K}$ then show that $\vec{a} + \vec{b} = \vec{a} - \vec{b}$ are perpendicular.

Sol $\vec{a} + \vec{b} = 6\hat{i} + 2\hat{j} - 8\hat{K}$

and $\vec{a} - \vec{b} = 4\hat{i} - 4\hat{j} + 2\hat{K}$

so, $(\vec{a} \cdot \vec{b}) \cdot (\vec{a} \cdot \vec{b}) = (5\hat{i} + 2\hat{j} - 8\hat{K}) \cdot (4\hat{i} - 4\hat{j} + 2\hat{K})$

or, $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 24 - 8 - 16 = 0$

Therefore, $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are perpendicular vectors.

Ans.